ENGINEERING
STUDY HUB

INTERNAL COMBUSTION ENGINES
PRINCIPLES, DESIGN, AND APPLICATIONS

POWERING INNOVATION

Otto cycle — brief overview

Introduction

The Otto cycle is the fundamental air-standard cycle used to model the operation of spark-ignition (SI) internal combustion engines. It provides a simplified thermodynamic framework to understand how energy is converted from the combustion of fuel into useful mechanical work. By idealizing the processes, the cycle allows engineers and students to analyze performance without the complexities of real engine behavior.

The cycle consists of four distinct processes: two isentropic (adiabatic and reversible) processes representing compression and expansion, and two constant-volume processes representing heat addition and heat rejection. Together, these processes form a closed loop on pressure–volume (P–V) and temperature–entropy (T–S) diagrams, illustrating the complete energy transformation within the engine cylinder.

The importance of the Otto cycle lies in its ability to predict the theoretical efficiency of SI engines. This efficiency depends primarily on the compression ratio of the engine and the specific heat ratio of the working fluid (air). Although real engines deviate from the ideal due to friction, heat losses, and incomplete combustion, the Otto cycle remains the cornerstone for understanding engine thermodynamics and serves as the baseline for comparing improvements in modern engine design.

Historical background of Otto cycle

Named after Nikolaus Otto, who pioneered the four-stroke internal combustion engine in the late 19th century. The theoretical cycle formalizes the main strokes—intake, compression, power, and exhaust—into an ideal thermodynamic model used for analysis and design of SI engines.

Applications (spark-ignition engines, petrol engines)

Predominantly used in petrol/gasoline engines across passenger cars, motorcycles, small generators, lawn equipment, and light aviation piston engines. Wherever rapid throttle response, lower unit weight, and simpler fuel systems are desired, SI engines modeled by the Otto cycle are common.

Assumptions of the air-standard cycle

- Working fluid is air that behaves as an ideal gas with constant specific heats.
- Cycle is closed; combustion is replaced by external heat addition at constant volume.
- All processes are internally reversible; compression and expansion are isentropic.
- No pumping, friction, or heat losses to surroundings; valves and combustion are idealized.

Under these assumptions, the Otto cycle thermal efficiency is:

$$\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}}$$

where $r = \frac{V_1}{V_2}$ is the compression ratio and $\gamma = \frac{c_p}{c_v}$.

Processes of the Otto Cycle

Isentropic compression (1–2)

In this stage, the piston moves upward from bottom dead center (BDC) to top dead center (TDC), compressing the air–fuel mixture trapped inside the cylinder. The process is assumed to be adiabatic and reversible, which means no heat is exchanged with the surroundings and entropy remains constant. As the volume decreases, both pressure and temperature rise significantly. This rise in temperature is crucial because it prepares the mixture for efficient combustion in the next stage. The work done on the mixture during compression is stored as internal energy, which will later be released during expansion. In real engines, some heat transfer and frictional losses occur, but in the ideal Otto cycle, these are neglected to simplify analysis.

Constant volume heat addition (2–3)

At the end of compression, the spark plug ignites the compressed air–fuel mixture. In the ideal cycle, this combustion is modeled as a constant volume heat addition process. Since the piston is momentarily at TDC, the volume remains fixed, but the rapid combustion releases a large amount of heat energy into the working fluid. This sudden energy release causes a sharp increase in both pressure and temperature inside the cylinder. On a P–V diagram, this process is represented by a vertical line, while on a T–S diagram, it appears as a horizontal shift to higher entropy. In reality, combustion takes a finite time and occurs slightly before TDC, but the constant volume assumption provides a close approximation for thermodynamic analysis.

Isentropic expansion (3–4)

This is the power stroke, the heart of the engine's operation. The high-pressure, high-temperature gases expand adiabatically and reversibly, pushing the piston downward from TDC to BDC. Because the process is isentropic, no heat is exchanged with the surroundings, and the entropy remains constant. The internal energy stored during compression and combustion is now converted into useful mechanical work delivered to the crankshaft. During this expansion, both pressure and temperature decrease as the volume increases. On the P–V diagram, this appears as a smooth downward curve, mirroring the compression process. This stage is where the engine produces its net positive work, making it the most important process in the cycle.

Constant volume heat rejection (4–1)

At the end of expansion, the piston reaches BDC, and the exhaust valve opens. In the ideal Otto cycle, this is modeled as a constant volume heat rejection process. The working fluid rejects heat to the surroundings while the piston is momentarily stationary, so the volume remains constant. As heat is removed, both pressure and temperature drop sharply, returning the system to its initial state. On the P–V diagram, this process is represented by a vertical line downward. In real engines, exhaust gases are expelled during the exhaust stroke, and heat transfer occurs continuously to the cooling system, but the constant volume assumption simplifies the analysis. This step completes the cycle, preparing the cylinder for the next intake and compression sequence.

Derivation of efficiency

Goal and definition: Thermal efficiency η_{otto} of the Otto cycle is the fraction of supplied heat converted into net work. By definition,

$$\eta_{otto} = \frac{W_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}.$$

For the air-standard Otto cycle (ideal gas, constant specific heats, internally reversible), heat transfer occurs only during the two constant-volume processes: heat addition $2 \to 3$ and heat rejection $4 \to 1$.

Heat quantities at constant volume: For a constant-volume process, the heat transfer equals the change in internal energy:

$$q = \Delta u = c_v\,(T_{\text{final}} - T_{\text{initial}}).$$

Therefore,

$$q_{in} = c_v\,(T_3 - T_2), \qquad q_{out} = c_v\,(T_4 - T_1).$$

Substituting into the efficiency definition gives

$$\eta_{otto} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_v\,(T_4 - T_1)}{c_v\,(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}.$$

This is the general expression before using isentropic relations.

Isentropic relations for compression and expansion: The processes $1 \to 2$ (compression) and $3 \to 4$ (expansion) are adiabatic and reversible (isentropic). For an ideal gas undergoing an isentropic process,

$$TV^{\gamma-1} = \text{const}, \qquad PV^{\gamma} = \text{const}, \qquad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}, \qquad T_4 = T_3\left(\frac{V_4}{V_3}\right)^{\gamma-1},$$

where $\gamma = \frac{c_p}{c_v}$ and $r = \frac{V_1}{V_2}$ is the compression ratio. In the ideal Otto cycle, the constant-volume processes occur at the minimum and maximum cylinder volumes, so $V_3 = V_2$ (at TDC) and $V_4 = V_1$ (at BDC). Hence,

$$T_2 = T_1\,r^{\gamma-1}, \qquad T_4 = \frac{T_3}{r^{\gamma-1}}.$$

Introducing the heat-addition ratio: Define the constant-volume heat addition ratio

$$\alpha = \frac{T_3}{T_2} \quad \Rightarrow \quad T_3 = \alpha\,T_2.$$

Using the isentropic expansion relation,

$$T_4 \;=\; \frac{T_3}{r^{\gamma-1}} \;=\; \frac{\alpha\, T_2}{r^{\gamma-1}}.$$

With $T_2 = T_1\, r^{\gamma-1}$, we obtain

$$T_4 \;=\; \frac{\alpha\,(T_1\, r^{\gamma-1})}{r^{\gamma-1}} \;=\; \alpha\, T_1.$$

Thus the four state temperatures can be expressed in terms of T_1, r, γ, and α:

$$T_2 = T_1\, r^{\gamma-1}, \quad T_3 = \alpha\, T_1\, r^{\gamma-1}, \quad T_4 = \alpha\, T_1.$$

Substituting into the efficiency expression:

$$\eta_{otto} \;=\; 1 - \frac{T_4 - T_1}{T_3 - T_2} \;=\; 1 - \frac{\alpha T_1 - T_1}{\alpha T_1 r^{\gamma-1} - T_1 r^{\gamma-1}} \;=\; 1 - \frac{(\alpha-1)T_1}{(\alpha-1)T_1 r^{\gamma-1}} \;=\; 1 - \frac{1}{r^{\gamma-1}}.$$

Importantly, the factor $(\alpha - 1)$, which represents the magnitude of heat addition at constant volume, cancels out. This shows that the ideal Otto efficiency depends only on the compression ratio r and the specific heat ratio γ, not on the amount of heat added (as long as the cycle remains Otto-type).

Key result and interpretation:

$$\boxed{\eta_{otto} \;=\; 1 - \frac{1}{r^{\gamma-1}}}$$

- Higher *compression ratio* r increases efficiency because the mixture is compressed to a higher temperature before combustion, improving the cycle's ability to convert heat into work.
- Higher *specific heat ratio* γ (typical of gases with fewer degrees of freedom) increases efficiency because adiabatic compression and expansion produce larger temperature swings for the same volume change.

Alternative derivation via work and heat: The net work per cycle is the area enclosed by the cycle on the P–V diagram:

$$W_{net} \;=\; q_{in} - q_{out} \;=\; c_v\,(T_3 - T_2) - c_v\,(T_4 - T_1).$$

Using the temperature relations above yields

$$W_{net} \;=\; c_v\left[\alpha T_1 r^{\gamma-1} - T_1 r^{\gamma-1} - \alpha T_1 + T_1\right] \;=\; c_v(\alpha-1)T_1\left(r^{\gamma-1} - 1\right).$$

Dividing by $q_{in} = c_v(\alpha-1)T_1 r^{\gamma-1}$ recovers

$$\eta_{otto} = \frac{W_{net}}{q_{in}} = \frac{r^{\gamma-1} - 1}{r^{\gamma-1}} = 1 - \frac{1}{r^{\gamma-1}}.$$

This route explicitly shows how the net work scales with $(\alpha - 1)$ but the *efficiency* does not.

Notes on assumptions and limitations: - *Air-standard model:* Treats the working fluid as air with constant c_p, c_v and γ; replaces real combustion with external heat addition at constant volume.
- *Isentropic compression/expansion:* Assumes adiabatic, reversible processes; real engines experience friction and heat loss, reducing actual efficiency.
- *Constant-volume idealization:* Real combustion is finite-rate and not perfectly at constant volume; however, the model is a good first approximation.
- *Implication:* While the formula shows that increasing r improves efficiency, SI engines are limited by knock, material constraints, and cooling requirements, so practical r values are bounded.

Factors Affecting Efficiency

Effect of compression ratio

The compression ratio $r = \frac{V_1}{V_2}$ is one of the most influential parameters in the Otto cycle. It represents how much the air–fuel mixture is compressed before ignition. From the efficiency expression

$$\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}},$$

it is clear that efficiency increases monotonically with higher compression ratio. This happens because a larger r means the mixture is compressed to a smaller volume, raising its pressure and temperature before combustion. As a result, the cycle extracts more useful work from the same amount of heat supplied.

Physically, when the compression ratio is higher:

- The temperature at the end of compression (T_2) is higher, which improves the effectiveness of combustion.
- The expansion stroke (3–4) starts at a higher pressure and temperature, allowing more work to be extracted during expansion.

- The fraction of heat rejected during the constant-volume cooling process (4–1) becomes smaller relative to the heat supplied, directly improving efficiency.

However, in real spark-ignition engines, the compression ratio cannot be increased indefinitely. Beyond a certain limit, the high temperature at the end of compression causes the fuel–air mixture to auto-ignite before the spark occurs, a phenomenon known as *knock*. Knock leads to pressure oscillations, reduced efficiency, and potential engine damage. Therefore, practical SI engines typically operate with compression ratios between 8:1 and 12:1, depending on fuel quality and engine design. Modern technologies such as variable compression ratio (VCR) engines and high-octane fuels are being developed to push this limit further.

Effect of specific heat ratio

The specific heat ratio $\gamma = \frac{c_p}{c_v}$ also plays a crucial role in determining efficiency. It appears in the exponent of the compression ratio in the efficiency formula:

$$\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}}.$$

A higher value of γ increases the efficiency for a given compression ratio. This is because a larger γ implies that the gas undergoes a greater temperature rise during compression and a greater temperature drop during expansion, enhancing the work output relative to the heat input.

The value of γ depends on the molecular structure of the working fluid:

- **Monatomic gases** (e.g., helium, argon) have $\gamma \approx 1.66$, which would yield very high efficiencies if they were practical working fluids.
- **Diatomic gases** like nitrogen and oxygen (the main components of air) have $\gamma \approx 1.4$ at room temperature, which is the effective value used in air-standard cycle analysis.
- **Polyatomic gases** (e.g., CO_2, steam) have lower γ values, around 1.2–1.3, leading to lower theoretical efficiencies.

In practice, the effective γ of air decreases slightly at high temperatures because additional molecular vibrational modes become active, increasing the specific heats. This means that the actual efficiency of real engines is somewhat lower than the ideal prediction. Since air has $\gamma \approx 1.4$, this sets a natural upper bound on the efficiency of spark-ignition engines, even if compression ratios are increased.

In summary:

- Increasing the **compression ratio** improves efficiency but is limited by knock and material constraints.
- A higher **specific heat ratio** improves efficiency, but since air is the working fluid, $\gamma \approx 1.4$ is the practical limit.

Together, these two parameters explain why the Otto cycle efficiency is fundamentally capped in real engines, and why engineers focus on advanced combustion strategies, better fuels, and alternative cycles to push beyond these limits.

Numerical Questions on Otto Cycle

Question 1

An air-standard Otto cycle has a compression ratio of $r = 8$. The air at the beginning of compression is at $P_1 = 1\,\text{bar},\ T_1 = 300\,\text{K}$. The maximum temperature during the cycle is $T_3 = 2000\,\text{K}$. Assume $\gamma = 1.4$ and $c_v = 0.718\,\text{kJ/kg·K}$.

Find:

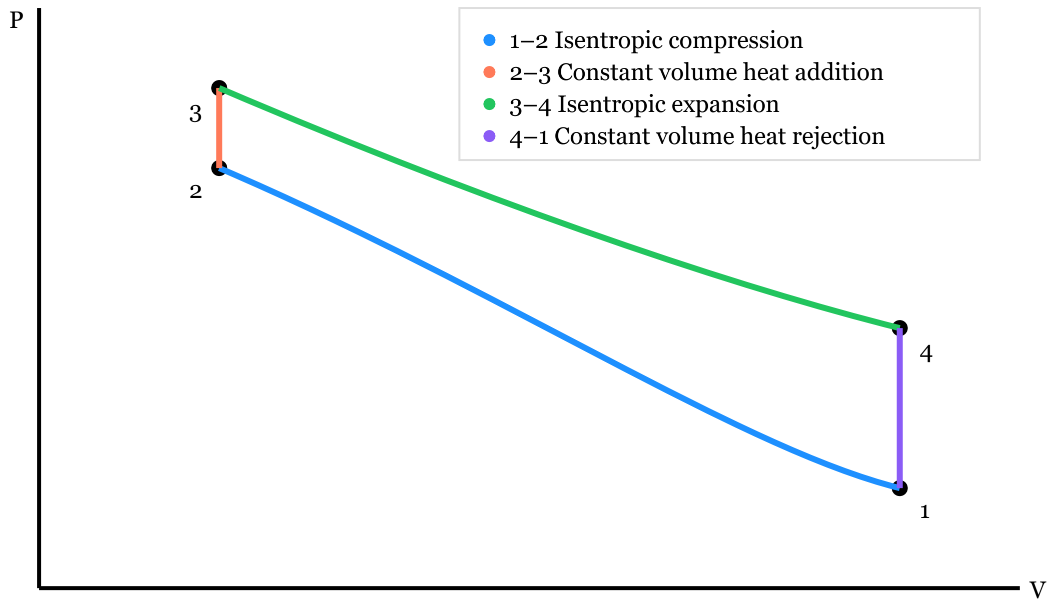
1. The temperatures at all four states (T_2, T_3, T_4).
2. The cycle efficiency.
3. The mean effective pressure (MEP).

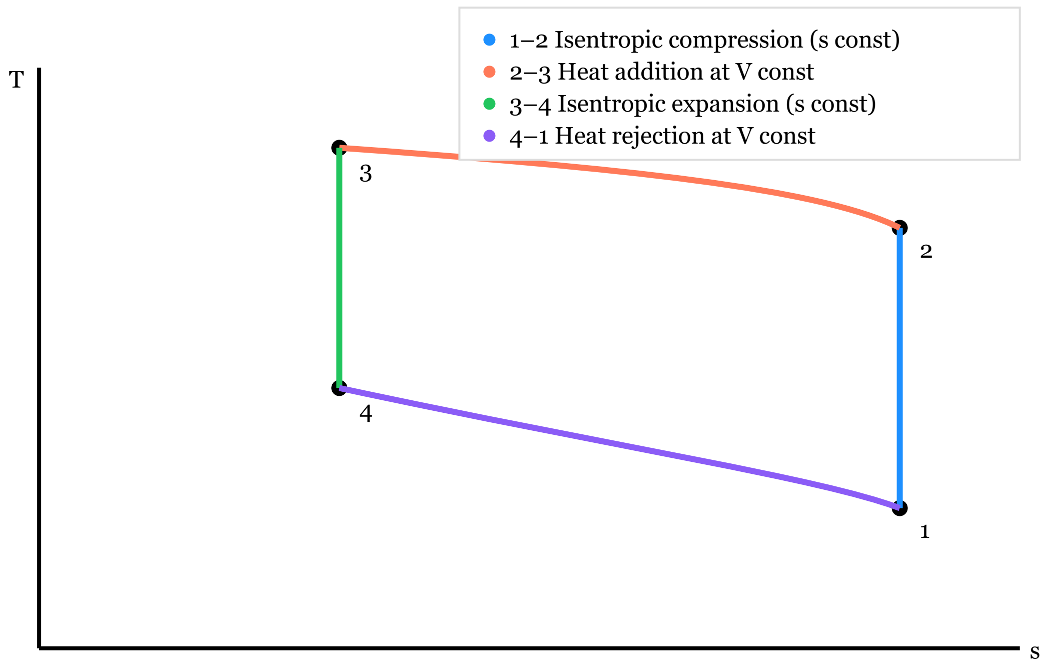
Question 2

In an ideal Otto cycle, the compression ratio is $r = 10$. The initial pressure and temperature are $P_1 = 1\,\text{bar},\ T_1 = 290\,\text{K}$. The heat added per unit mass during constant volume combustion is $q_{in} = 1800\,\text{kJ/kg}$. Take $c_v = 0.718\,\text{kJ/kg·K},\ R = 0.287\,\text{kJ/kg·K},\ \gamma = 1.4$.

Find:

1. The maximum pressure in the cycle.
2. The net work output per kg of air.
3. The thermal efficiency of the cycle.





Practical limitations

- **Knock:** Excessive compression ratio in spark-ignition engines leads to auto-ignition of the fuel–air mixture, causing knock and limiting r.
- **Material strength:** Higher pressures and temperatures demand stronger, costlier engine materials.

- **Cooling requirements:** Real engines lose heat to cylinder walls and require cooling systems, reducing actual efficiency compared to the ideal cycle.